Mathematics General 2

FORMULAE AND DATA SHEET

Financial Mathematics

Simple interest

$$I = Prn$$

- P is initial amount
- r is interest rate per period, expressed as a decimal
- n is number of periods

Compound interest

$$A = P\left(1 + r\right)^n$$

- A is final amount
- P is initial amount
- r is interest rate per period, expressed as a decimal
- n is number of compounding periods

Present value and future value

$$PV = \frac{FV}{\left(1 + r\right)^n}, \qquad FV = PV\left(1 + r\right)^n$$

- r is interest rate per period, expressed as a decimal
- n is number of compounding periods

Straight-line method of depreciation

$$S = V_0 - Dn$$

- S is salvage value of asset after n periods
- V_0 is initial value of asset
- D is amount of depreciation per period
- n is number of periods

Declining-balance method of depreciation

$$S = V_0\left(1 - r\right)^n$$

- S is salvage value of asset after n periods
- V_0 is initial value of asset
- r is depreciation rate per period, expressed as a decimal
- n is number of periods

Data Analysis

Mean of a sample

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

z-score

For any score x,

$$z = \frac{x - \bar{x}}{s}$$

- $\bar{x}$ is mean
- s is standard deviation

Outlier(s)

score(s) less than $Q_L - 1.5 \times IQR$

or

score(s) more than $Q_U + 1.5 \times IQR$

- Q_L is lower quartile
- Q_U is upper quartile
- IQR is interquartile range

Least-squares line of best fit

$$y = \text{gradient} \times x + y\text{-intercept}$$

$$\text{gradient} = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

$$y\text{-intercept} = \bar{y} - \left(\text{gradient} \times \bar{x}\right)$$

- r is correlation coefficient
- $\bar{x}$ is mean of x scores
- $\bar{y}$ is mean of y scores

Normal distribution

- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

Spherical Geometry

Circumference of a circle

$$C = 2\pi r \quad \text{or} \quad C = \pi D$$

r is radius

D is diameter

Arc length of a circle

$$l = \frac{\theta}{360} 2\pi r$$

r is radius

θ is number of degrees in central angle

Radius of Earth

(taken as) 6400 km

Time differences

For calculation of time differences using longitude:

$15^\circ = 1$ hour time difference

Area

Circle

$$A = \pi r^2$$

r is radius

Sector

$$A = \frac{\theta}{360} \pi r^2$$

r is radius

θ is number of degrees in central angle

Annulus

$$A = \pi\left(R^2 - r^2\right)$$

R is radius of outer circle

r is radius of inner circle

Trapezium

$$A = \frac{h}{2}\left(a + b\right)$$

h is perpendicular height

a and b are the lengths of the parallel sides

Area of land and catchment areas

unit conversion: $1 \text{ ha} = 10\,000 \text{ m}^2$

Surface Area

Sphere

$$A = 4\pi r^2$$

r is radius

Closed cylinder

$$A = 2\pi r^2 + 2\pi r h$$

r is radius

h is perpendicular height

Volume

Prism or cylinder

$$V = Ah$$

A is area of base

h is perpendicular height

Pyramid or cone

$$V = \frac{1}{3} Ah$$

A is area of base

h is perpendicular height

Volume and capacity

unit conversion: $1 \text{ m}^3 = 1000 \text{ L}$

Approximation Using Simpson's Rule

Area

$$A \approx \frac{h}{3}\left(d_f + 4d_m + d_l\right)$$

h is distance between successive measurements

d_f is first measurement

d_m is middle measurement

d_l is last measurement

Volume

$$V \approx \frac{h}{3}\left\{A_L + 4A_M + A_R\right\}$$

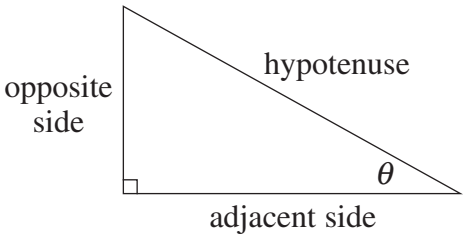
h is distance between successive measurements

A_L is area of left end

A_M is area of middle

A_R is area of right end

Trigonometric Ratios



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine rule

In $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

In $\triangle ABC$,

$$A = \frac{1}{2}ab\sin C$$

Cosine rule

In $\triangle ABC$,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Units of Memory and File Size

1 byte = 8 bits

1 kilobyte = 2^{10} bytes = 1024 bytes

1 megabyte = 2^{20} bytes = 1024 kilobytes

1 gigabyte = 2^{30} bytes = 1024 megabytes

1 terabyte = 2^{40} bytes = 1024 gigabytes

Blood Alcohol Content Estimates

$$BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M}$$

or

$$BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

N is number of standard drinks consumed

H is number of hours of drinking

M is person's mass in kilograms

Distance, Speed and Time

$$D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S}$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{stopping distance} = \left\{\begin{matrix}\text{reaction-time}\\ \text{distance}\end{matrix}\right\} + \left\{\begin{matrix}\text{braking}\\ \text{distance}\end{matrix}\right\}$$

Probability of an Event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Straight Lines

Gradient

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form

$$y = mx + b$$

m is gradient

b is y-intercept

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